CS/MATH 3414 — Numerical Methods

Fall 2026

Instructor: Sumeet Khatri

Tuesday, August 25, 2026

Basic Information

- **Timing and location:** 11am to 12:15pm in McBryde Hall 316
- **TA:** Abdullah Al Noman
- **Office hours:**
  - Sumeet: Tuesdays from 4pm to 5pm on Zoom (link in Canvas). Also available via appointment; please send me an email.
  - Noman: Mondays, Wednesdays, Fridays from 3pm to 5pm in Torgersen 1120.

Numerical Methods

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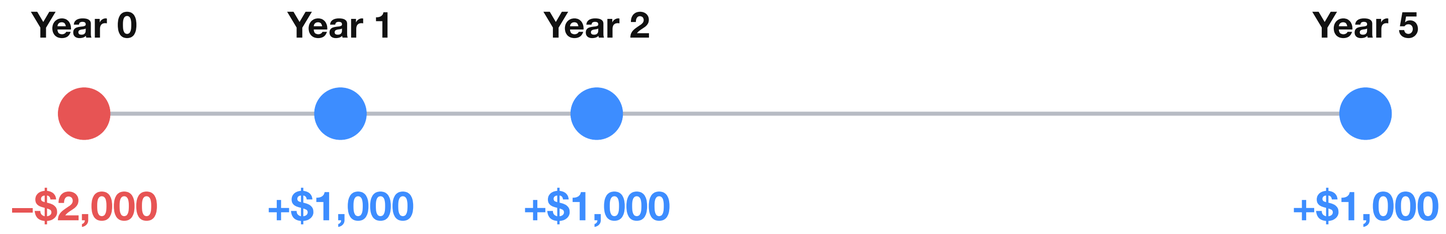
- Third- and fourth-order equations can also be exactly solved by hand using formulas. But this changes starting from the fifth order! We must resort to **approximate/numerical methods**!

Suppose you invest $2,000 now and later receive $1,000 in years 1, 2, and 5 (nothing in years 3 and 4).



Question: What annual rate of return makes the future payments worth exactly $2,000 today?

Money today is worth more than the same money in the future.

An amount x received in t years has present value of $\frac{x}{(1+r)^t}$.

r is the desired *internal rate of return*.

We are interested in the *break-even rate* where your future money gains equal your starting costs.

Let $d = \dfrac{1}{1+r}$ be the *discount factor*.

An amount x received in t years has present value of $\dfrac{x}{(1+r)^t} = xd^t$.

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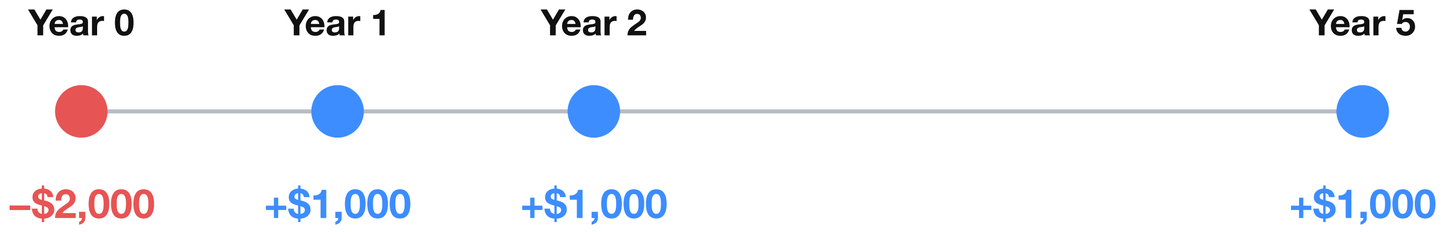

The break-even equation is then $1000d + 1000d^2 + 1000d^5 = 2000$

Divide by 1000 on both sides $\Rightarrow d + d^2 + d^5 - 2 = 0$.

$$d^5 + d^2 + d - 2 = 0$$

This cannot be solved analytically with a formula!
We must resort to **approximate numerical methods**!

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Microsoft Excel uses exactly such an approximate numerical method!

We will see exactly what method is used to solve such equations.

IRR function

▸ *Applies To*

This article describes the formula syntax and usage of the **IRR** function in Microsoft Excel.

Description

Returns the internal rate of return for a series of cash flows represented by the numbers in values. These cash flows do not have to be even, as they would be for an annuity. However, the cash flows must occur at regular intervals, such as monthly or annually. The internal rate of return is the interest rate received for an investment consisting of payments (negative values) and income (positive values) that occur at regular periods.

Syntax

IRR(values, [guess])

The IRR function syntax has the following arguments:

Values: Required. An array or a reference to cells that contain numbers for which you want to calculate the internal rate of return.

- Values must contain at least one positive value and one negative value to calculate the internal rate of return.
- IRR uses the order of values to interpret the order of cash flows. Be sure to enter your payment and income values in the sequence you want.
- If an array or reference argument contains text, logical values, or empty cells, those values are ignored.

Guess: Optional. A number that you guess is close to the result of IRR.

- Microsoft Excel uses an iterative technique for calculating IRR. Starting with guess, IRR cycles through the calculation until the result is accurate within 0.00001 percent. If IRR can't find a result that works after 20 tries, the #NUM! error value is returned.
- In most cases you do not need to provide guess for the IRR calculation. If guess is omitted, it is assumed to be 0.1 (10 percent).
- If IRR gives the #NUM! error value, or if the result is not close to what you expected, try again with a different value for guess.

Fitting a line to data

Determining trends in data

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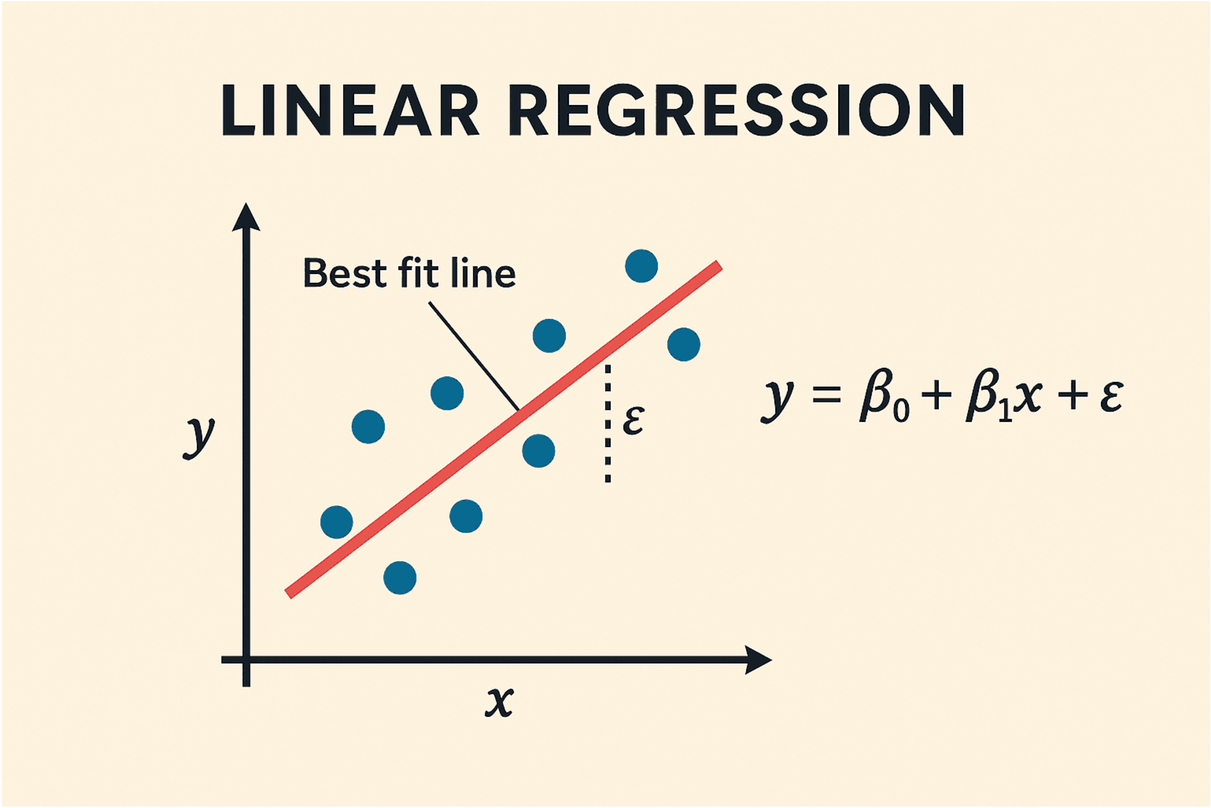

https://www.statisticalaid.com/linear-regression/

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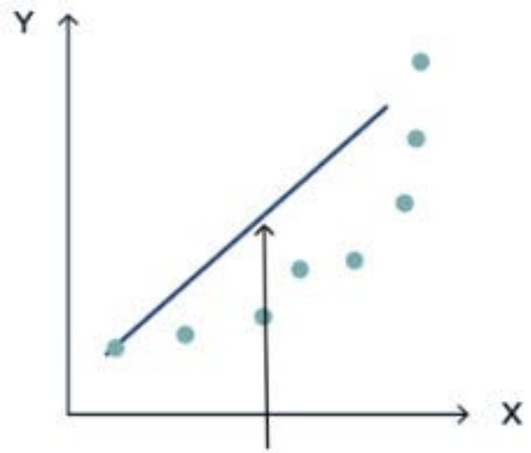
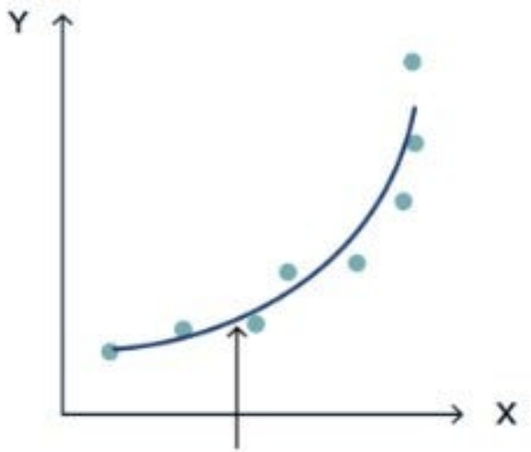
Simple linear model



$y=b_0+b_1x$

Polynomial model



$y=b_0+b_1x+b_2x_1^2$

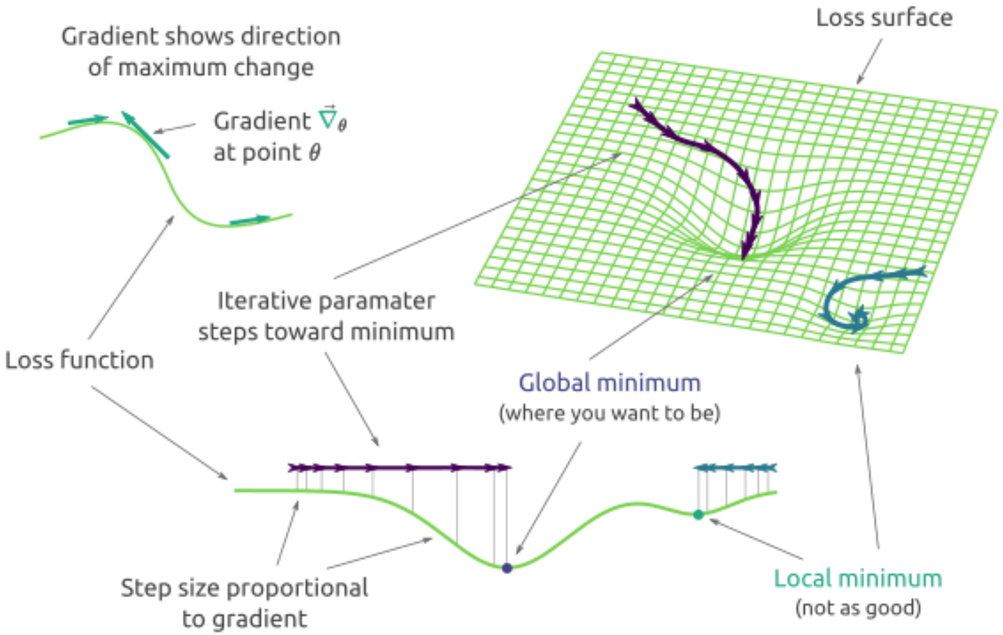
Machine Learning

Making predictions based on training data

https://harvard-edge.github.io/cs249r_book_dev/vol1/nn_computation/nn_computation.html

Optimization

- Finding the maximum or minimum value of a function subject to constraints.
- The function and constraints encode something about the problem of interest.
- Many software packages for solving (MATLAB, Python packages like SciPy, Mathematica, etc.).
- We will understand some of these methods! e.g., **gradient descent** for unconstrained problems.



General overview of topics

1. **Representation of real numbers on computers:** floating-point numbers; significant digits; accuracy and precision; error analysis of algorithms.

2. **Numerical solutions to single-variable (non-)linear equations:** Newton's method, secant method; convergence analysis and guarantees

3. **Computational linear algebra:** matrix multiplication, singular-value decomposition, linear systems, Gaussian elimination; least-squares/linear regression; polynomial interpolation.

4. **Simulating random processes:** random numbers, sampling from probability distributions, Monte Carlo simulations.

5. **Optimization:** gradient descent, neural networks, linear programming and reinforcement learning

AI Usage

- Allowed as a learning tool
- You can query things related to the course material, like:
  - “Explain what a floating-point number is.”
  - “What are the different methods for fitting a line to given data points?”
  - “How does Newton’s method work? Provide references.”
- **Remember:** AI can make mistakes! Being able to identify mistakes, and being skeptical of AI in general, is the purpose of learning things in a formal, instructional setting like this one.
  - AI is most effective for you *after* you have already learned the basic material!

Assessments

- Six in-class assignments
  - To be completed during the class time (75 minutes) and submitted by the end of class.
  - You are allowed to consult with peers and with me.
  - AI use is not allowed!
- One final exam on the final day of class, December 8.

Grading

Final grade component	Scheme 1	Scheme 2	Scheme 3	Scheme 4
In-class assignments (top five)	60%	50%	30%	0%
Final exam	30%	50%	60%	100%
Participation	10%	0%	10%	0%

Your final grade will be given by whichever among the four schemes gives you the highest grade. You do not have to pick a scheme!