CS 4134 – Quantum Computation and Information Processing

In-class practice problems II

Virginia Tech Department of Computer Science, Spring 2026
Instructor: Sumeet Khatri

Name:

Date: Tuesday, April 28, 2026 (in class)

Due: End of class!

Number of problems: 5

Instructions: These problems are for BONUS points that will count towards your score on Exam 4, if you submit by the end of class. You are not required to complete all problems. The number of bonus points you get will be based on how many problems you complete, and the quality of the solutions. You may work with others to solve the problems, and you may ask the instructor for help as well. There is extra paper available if you need it.

For the grader only

Problem	1	2	3	4	5	**Final Score**
Score						

1. Let $|v_1\rangle, |v_2\rangle \in \mathbb{C}^d$ be arbitrary vectors, and let $M \in \mathrm{L}(\mathbb{C}^d)$ be an arbitrary linear operator (i.e., $d \times d$ matrix). Using the definition of the trace of a linear operator, prove the following identities.

   (a) $\mathrm{Tr}\big[|v_1\rangle\langle v_2|\big] = \langle v_2|v_1\rangle$

   (b) $\mathrm{Tr}\big[M|v_1\rangle\langle v_2|\big] = \langle v_2|M|v_1\rangle$

2. Let U be an arbitrary 2×2 unitary matrix. Prove that $(U \otimes U)|\Psi^-\rangle\langle\Psi^-|(U \otimes U)^\dagger = |\Psi^-\rangle\langle\Psi^-|$.

3. (a) State the definition of a separable state and an entangled state.

   (b) State the definition of the Schmidt decomposition of a bipartite state vector $|\psi\rangle_{AB}$.

   (c) Under what condition(s), based on the Schmidt decomposition, is $|\psi\rangle_{AB}$ entangled?

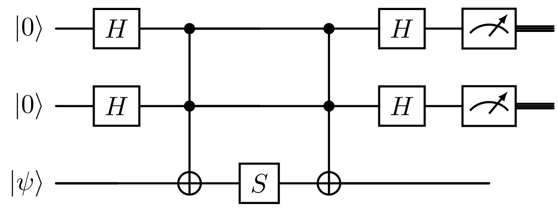
   (d) Based on the Schmidt decomposition, determine (with justification) whether or not the following state vectors are entangled.

      i. $\frac{1}{\sqrt{3}}|0\rangle|+\rangle + \sqrt{\frac{2}{3}}|1\rangle|-\rangle$.

      ii. $\frac{1}{4}\big(3|0,0\rangle - \sqrt{3}|0,1\rangle + \sqrt{3}|1,0\rangle - 2|1,1\rangle\big)$.

4. Draw the quantum circuit for the quantum teleportation protocol. State the definition of every gate and/or measurement you draw in the circuit.

5. Consider the following circuit:



The state vector $|\psi\rangle$ is arbitrary, and the final measurement is in the Pauli-Z basis. Show that the probability of both measurement outcomes being 0 is $\frac{5}{8}$.