Frontiers of Quantum Information and Computation

Sumeet Khatri

January 21, 2025

What is this course about?

- Understanding the basics of quantum information and computation
- Charting the developments over the last approx. 30 years
- Where the field is currently, and what is the path forward
- Uncover and explore ideas for future research

Course format

- 3/4 = lectures by me.
- 1/4 = presentations on recent research papers – I will provide suggestions on papers throughout the lectures.
- Course grading: just show up and ask lots of questions!
- Prerequisites: background in linear algebra is strongly recommended. But we will cover the basics!

Course outline

- **Part I: Basics of quantum information and computing**
  How do we describe the quantum world?

- **Part II: Extracting and exploiting information from the quantum world**
  How to extract classical information from quantum systems?

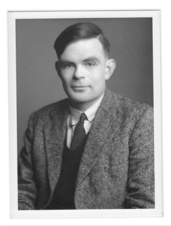
- **Part III: Quantum information processing over large distances**
  How can we perform quantum information processing and computing over large distances?

Recommended texts

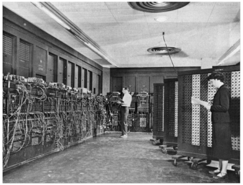
- ⋆ *Quantum Computation and Quantum Information*, by Michael Nielsen and Isaac Chuang.
- ⋆ *An Introduction to Quantum Computing, by Phillip Kaye*, Raymond Laflamme and Michele Mosca.
- ⋆⋆ *Introduction to Classical and Quantum Computing*, by Thomas G. Wong. **(Nice and gentle.)**
- *Quantum Information: From Foundations to Quantum Technology Applications*, by Dagmar Bruss and Gerd Leuchs (editors). **(Very broad in scope, discusses applications.)**
- *The Theory of Quantum Information*, by John Watrous.
- *Classical and Quantum Computation*, by A. Yu. Kitaev, A. H. Shen, M. N. Vyalyi. **(Quite advanced!)**
- *Principles of Quantum Communication Theory: A Modern Approach*, by Sumeet Khatri and Mark M. Wilde. Available at `https://arxiv.org/abs/2011.04672`.
- *An Introduction to Computational Learning Theory*, by Michael Kearns and Umesh Vazirani.

Other information

- I will provide some **recommended exercises** throughout.
- I will provide suggestions for research papers for final presentations. Feel free to think of your own!
- For a hands-on experience with some of the topics: you can use the **QuTIpy package**: `https://github.com/sumeetkhatri/QuTIpy`. If you are interested in contributing to the package, please let me know!



Turing



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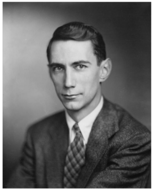
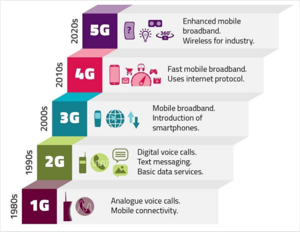


Shannon



Quantum mechanics

- In short, the study of matter on small scales.

Quantum mechanics

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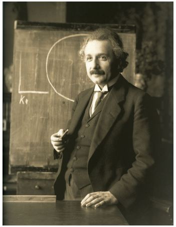
Foundational discoveries

Planck
Blackbody radiation
$E = h\nu$
(Discrete energies)

Heisenberg
$\Delta X \Delta P \geq \frac{\hbar}{2}$
(Uncertainty relation)

Schrödinger
$i\hbar \frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle$
(Dynamics)



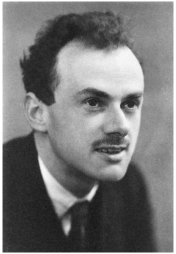
Einstein
Photoelectric effect
EPR paradox
(Entanglement)

Born
Born rule
(Measurements)

Quantum mechanics

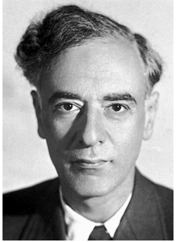
- In short, the study of matter on small scales.

Axiomatic mathematical formulation



Dirac

von Neumann



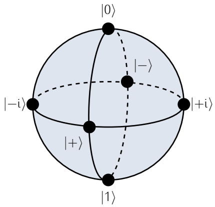
Landau

We will go through this in detail later!

- View physical, quantum-mechanical systems as *carriers of information*.

Bit

0 or 1;
or probabilistic mixture



Qubit

$|0\rangle$ or $|1\rangle$;
or *superposition*

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle,$$
$$\alpha, \beta \in \mathbb{C},$$
$$|\alpha|^2 + |\beta|^2 = 1$$

$$|\pm\rangle = \tfrac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$
$$|\pm\mathrm{i}\rangle = \tfrac{1}{\sqrt{2}}(|0\rangle \pm \mathrm{i}|1\rangle)$$

Abstract picture:

- Quantum system (atoms, photons, etc.) $\equiv$ Hilbert space.
- Physical evolution $\equiv$ quantum gate/quantum channel.

Quantum mechanics → quantum information

No-cloning theorem

Foundations of Physics, Vol. 1, No. 1, 1970

The Concept of Transition in Quantum Mechanics

James L. Park

Department of Physics, Washington State University, Pullman, Washington

Nature Vol. 299 28 October 1982

A single quantum cannot be cloned

W. K. Wootters*

Center for Theoretical Physics, The University of Texas at Austin, Austin, Texas 78712, USA

W. H. Zurek

Theoretical Astrophysics 130–33, California Institute of Technology, Pasadena, California 91125, USA

Volume 92A, number 6 PHYSICS LETTERS 22 November 1982

COMMUNICATION BY EPR DEVICES

D. DIEKS

Fysisch Laboratorium, Rijksuniversiteit Utrecht, Utrecht, The Netherlands

Received 17 August 1982
Revised manuscript received 21 September 1982

A recent proposal to achieve faster-than-light communication by means of an EPR-type experimental set-up is examined. We demonstrate that such superluminal communication is not possible. The crucial role of the linearity of the quantum mechanical evolution laws in preventing causal anomalies is stressed.

There does not exist a unitary (linear) operation that can copy an arbitrary quantum state.

Bell's theorem

Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(*Received* 4 *November* 1964)

Correlations in quantum mechanics do not generally correspond to a local-hidden-variable model.

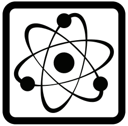
Nobel Prize 2022!

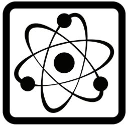
Entanglement

Alice and Bob images from `https://levelup.gitconnected.com/quantum-key-distribution-for-everyone-f08dd5646f33`

Entanglement

Alice





Bob

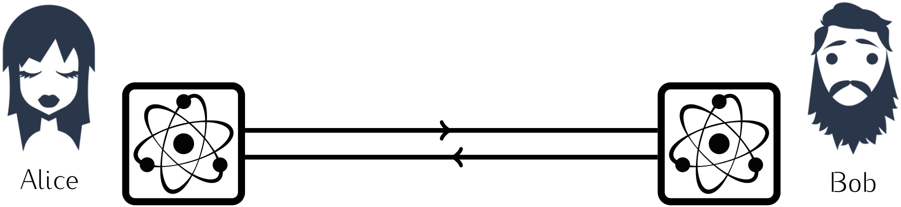
$$\rho_{AB} = \sigma_A \otimes \tau_B$$

Product state

Alice and Bob individually prepare their systems.

Alice and Bob images from **`https://levelup.gitconnected.com/quantum-key-distribution-for-everyone-f08dd5646f33`**

Entanglement



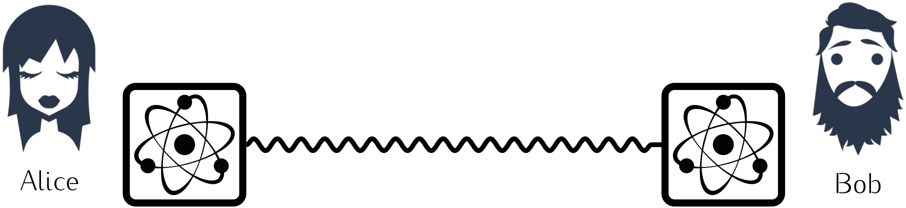
$$\rho_{AB} = \sum_{x \in \mathcal{X}} p(x) \sigma_A^x \otimes \tau_B^x$$

Separable state

Alice and Bob individually prepare their systems via local operations and classical communication.

Alice and Bob images from https://levelup.gitconnected.com/quantum-key-distribution-for-everyone-f08dd5646f33

Entanglement

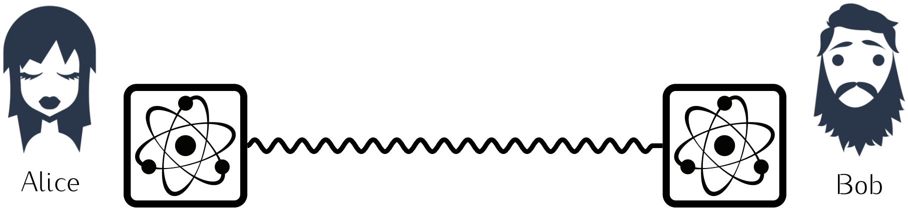


$$\rho_{AB} \neq \sum_{x \in \mathcal{X}} p(x) \sigma_A^x \otimes \tau_B^x$$

Entangled state

Correlations between Alice and Bob are non-local.
State of the individual systems not sufficient to describe the pair.

Alice and Bob images from `https://levelup.gitconnected.com/quantum-key-distribution-for-everyone-f08dd5646f33`



$$|\Phi^{\pm}\rangle_{AB} = \frac{1}{\sqrt{2}}(|0,0\rangle_{AB} \pm |1,1\rangle_{AB}), \quad |\Psi^{\pm}\rangle_{AB} = \frac{1}{\sqrt{2}}(|0,1\rangle_{AB} \pm |1,0\rangle_{AB})$$

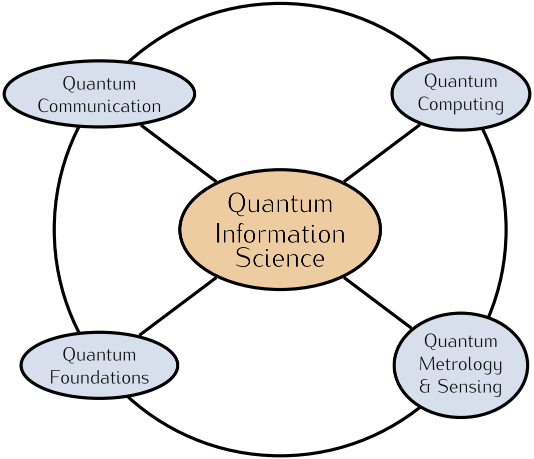
Bell states

State is locally random,
but joint measurement results are correlated.

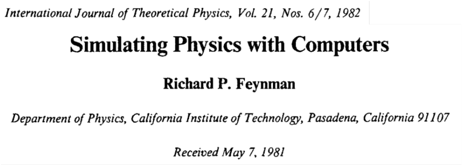
Alice and Bob images from https://levelup.gitconnected.com/quantum-key-distribution-for-everyone-f08dd5646f33

Quantum information science and quantum technologies

- Harness superposition and entanglement to do certain tasks better/faster. (e.g., factoring, simulation)
- Also discover *new* things. (e.g., teleportation, QKD)



Quantum computing

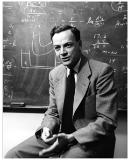

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

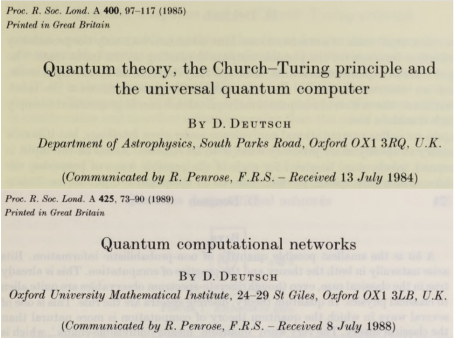
Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981




Proc. R. Soc. Lond. A **400**, 97–117 (1985)
Printed in Great Britain

Quantum theory, the Church–Turing principle and the universal quantum computer

By D. Deutsch

Department of Astrophysics, South Parks Road, Oxford OX1 3RQ, U.K.

(*Communicated by R. Penrose, F.R.S. – Received* 13 *July* 1984)

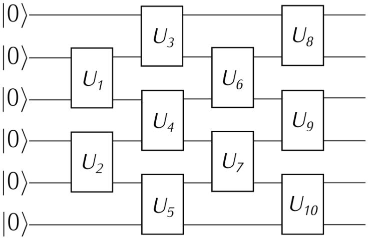
Proc. R. Soc. Lond. A **425**, 73–90 (1989)
Printed in Great Britain

Quantum computational networks

By D. Deutsch

Oxford University Mathematical Institute, 24–29 St Giles, Oxford OX1 3LB, U.K.

(*Communicated by R. Penrose, F.R.S. – Received* 8 *July* 1988)



- ⋆ Potential for speedups in simulating large quantum systems.
- ⋆ Known speedups for factoring (Shor) and database search (Grover).

Quantum Complexity Theory

(Preliminary Abstract) (1993)

Ethan Bernstein*
Umesh Vazirani†
Computer Science Division
University of California, Berkeley
Berkeley, CA 94720

Quantum computing

Algorithms for Quantum Computation: Discrete Logarithms and Factoring

(1994)

Peter W. Shor
AT&T Bell Labs
Room 2D-149
600 Mountain Ave.
Murray Hill, NJ 07974, USA

SIAM J. Comput.
Vol. 26, No. 5, pp. 1484–1509, October 1997

POLYNOMIAL-TIME ALGORITHMS FOR PRIME FACTORIZATION AND DISCRETE LOGARITHMS ON A QUANTUM COMPUTER*

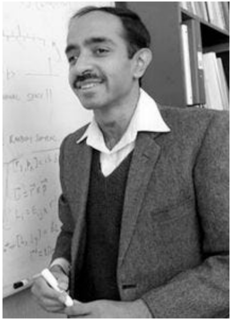
PETER W. SHOR†

Prime factorization in polynomial time!

A fast quantum mechanical algorithm for database search

Lov K. Grover
3C-404A, AT&T Bell Labs
600 Mountain Avenue
Murray Hill NJ 07974
lkg@mhcnet.att.com

(1996)



Quadratic speedup for search problems!

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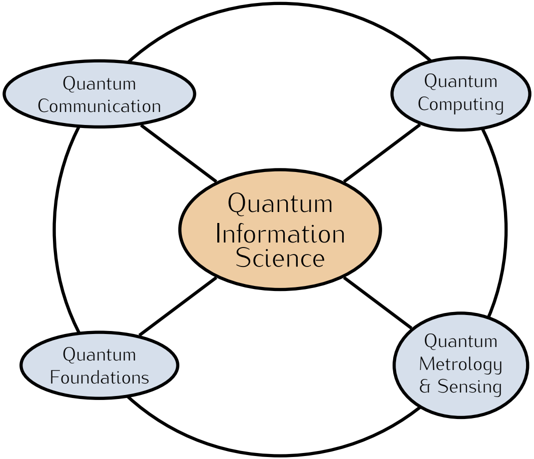
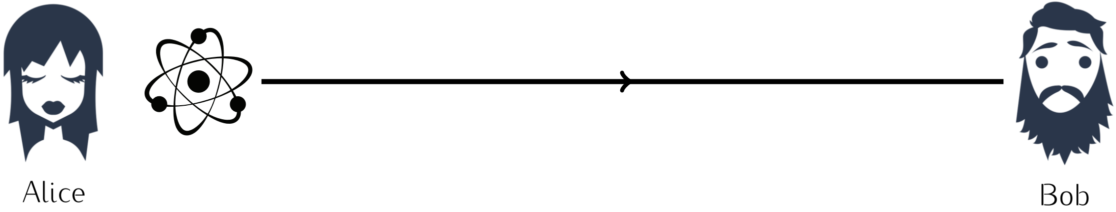


Quantum communication

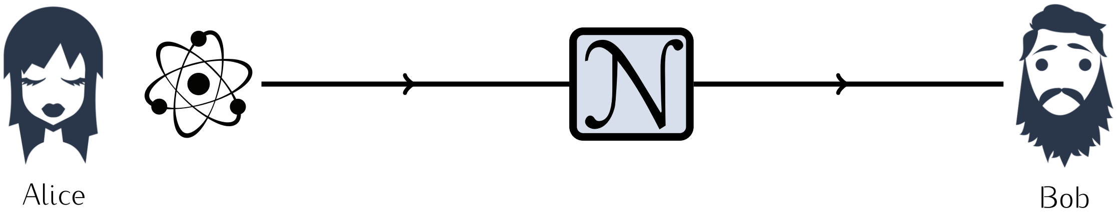
⋆ Using quantum systems and quantum strategies to send bits and qubits.



Ideal quantum channel from Alice to Bob.

Quantum communication

⋆ Using quantum systems and quantum strategies to send bits and qubits.



Noisy quantum channel from Alice to Bob, models imperfections in the transmission medium.

Teleportation: shared entanglement + classical communication = transmission of an arbitrary quantum state

PHYSICAL REVIEW
LETTERS

VOLUME 70 | 29 MARCH 1993 | NUMBER 13

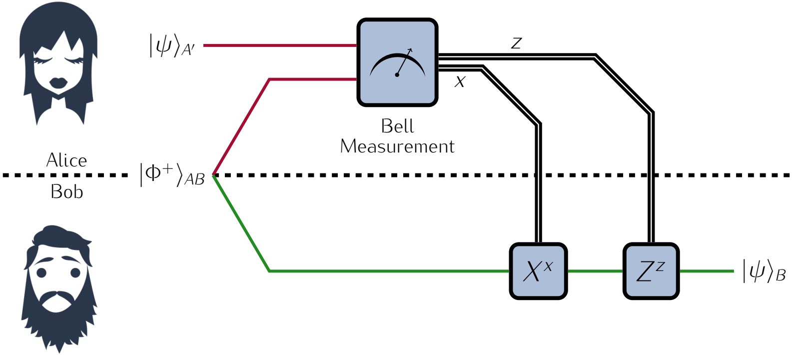
Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels

Charles H. Bennett,[1] Gilles Brassard,[2] Claude Crépeau,[2],[3]
Richard Jozsa,[2] Asher Peres,[4] and William K. Wootters[5]

No physical transmission of quantum systems! Only classical communication!

Quantum communication

Teleportation: shared entanglement + classical communication = transmission of an arbitrary quantum state



No physical transmission of quantum systems! Only classical communication!

Quantum communication

Quantum key distribution: private classical communication with quantum strategies/resources

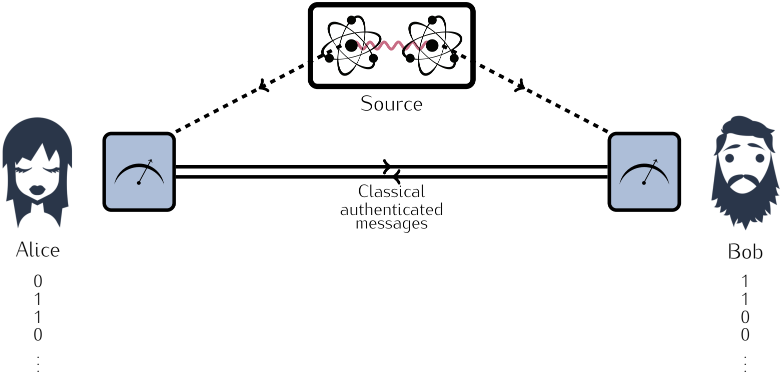
QUANTUM CRYPTOGRAPHY: PUBLIC KEY DISTRIBUTION AND COIN TOSSING

Charles H. Bennett (IBM Research, Yorktown Heights NY 10598 USA)
Gilles Brassard (dept. IRO, Univ. de Montreal, H3C 3J7 Canada)

International Conference on Computers, Systems & Signal Processing Bangalore, India December 10-12, 1984

Quantum communication

Quantum key distribution: private classical communication with quantum strategies/resources



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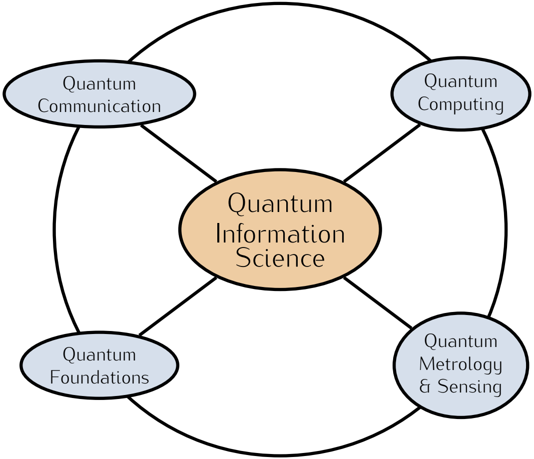


⋆ A precursor to quantum (machine) learning!

Quantum Detection and Estimation Theory

Carl W. Helstrom[1]

Received March 20, 1969

JOURNAL OF MULTIVARIATE ANALYSIS 3, 337–394 (1973)

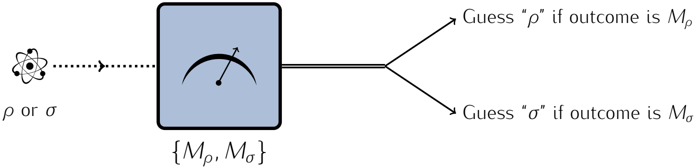
Statistical Decision Theory for Quantum Systems

A. S. Holevo

Steklov Mathematical Institute, Academy of Sciences of the U.S.S.R., Moscow, U.S.S.R.

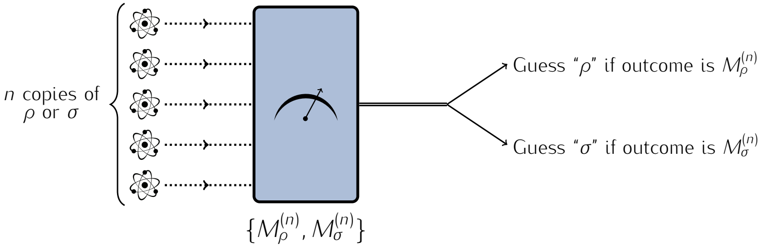
Communicated by Yu. A. Rozanov

★ A precursor to quantum (machine) learning!



Quantum hypothesis testing and estimation theory

★ A precursor to quantum (machine) learning!



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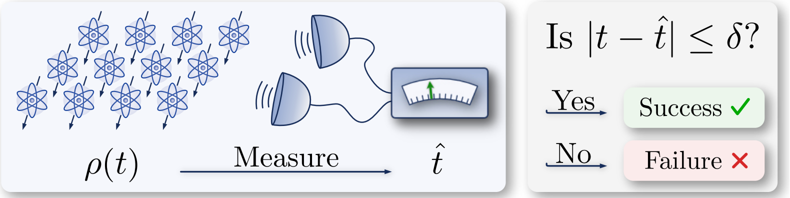
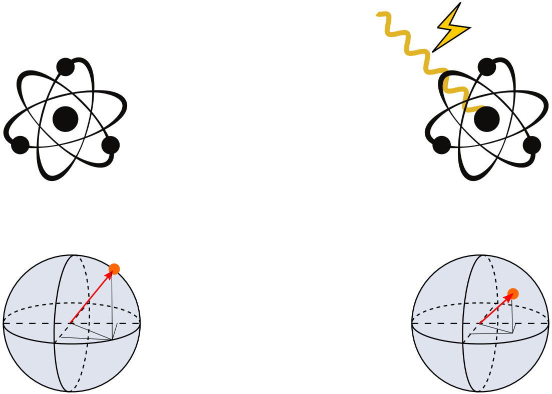


More recent developments

- ⋆ Making actual qubits is *hard*.
- ⋆ Theoretical protocols do not perform as expected – there are errors!

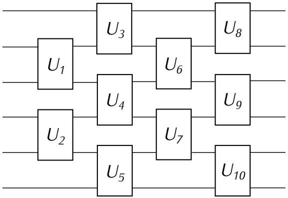
The problem: noise!



- ⋆ Causes of noise: interaction with the environment, imperfections in the medium.
- ⋆ Mathematical model: completely positive trace-preserving map (comes from the Schrödinger equation).
- ⋆ Noise means that theoretical schemes do not necessarily work exactly as described in practice!

Consequences of noise for quantum computation

- ⋆ So far, we assumed that every gate U_i is unitary.
- ⋆ In practice, there are two types of errors:
  - ▶ Coherent errors: $\widetilde{U}_i$ instead of U_i.
  - ▶ Incoherent errors: A *quantum channel* $\mathcal{N}_i$ instead of U_i



⋆ **How much noise can we tolerate while still doing a useful computation?**

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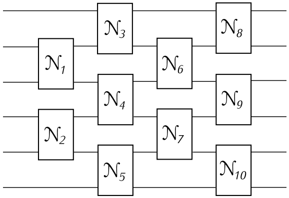


⋆ **How much noise can we tolerate while still doing a useful computation?**

Fighting noise and errors

RAPID COMMUNICATIONS

PHYSICAL REVIEW A

ATOMIC, MOLECULAR, AND OPTICAL PHYSICS

THIRD SERIES, VOLUME 52, NUMBER 4 OCTOBER 1995

RAPID COMMUNICATIONS

Scheme for reducing decoherence in quantum computer memory

Peter W. Shor*
AT&T Bell Laboratories, Room 2D-149, 600 Mountain Avenue, Murray Hill, New Jersey 07974

(Received 17 May 1995)

Fault-Tolerant Quantum Computation

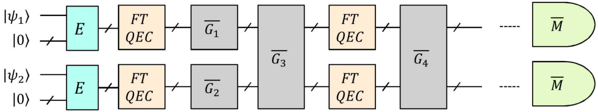
Peter W. Shor
AT&T Research
Room 2D-149
600 Mountain Ave.
Murray Hill, NJ 07974, USA
shor@research.att.com

Fighting noise and errors

- Analogous to (classical) error correction, $0 \mapsto 000$, $1 \mapsto 111$.

$$|0\rangle \mapsto |0,0,0\rangle, \; |1\rangle \mapsto |1,1,1\rangle.$$

- Fault-tolerant schemes allow for error-free computation, in principle.
- But this requires many good-quality qubits: ~ 1000 physical qubits per logical qubit.



From [https://www.osti.gov/servlets/purl/1640593]

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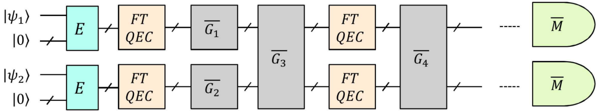

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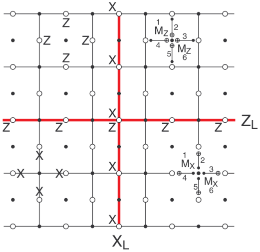

FIG. 1 (color online). A surface code logical qubit. Stabilizers $ZZZZ$ ($XXXX$) are associated with the data qubits (open circles) around each face (vertex). Syndrome qubits (dots) measure stabilizers using the indicated sequences of gates. Logical operators Z_L, X_L connect opposing boundaries.

From [PRA **86**, 032324 (2012)]

Fighting noise and errors

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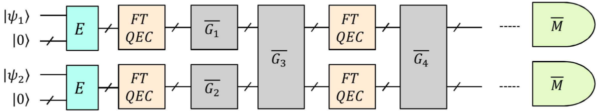
- We currently have:
  - Limited number of qubits (≈ 100).
  - Limited connectivity between qubits.
  - Limited circuit depth due to noise.



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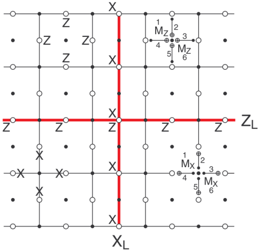

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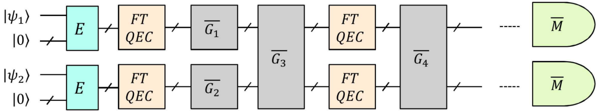
  - Limited circuit depth due to noise.

⋆ **Much current work is devoted to discovering and implementing error-correcting codes!**



From [https://www.osti.gov/servlets/purl/1640593]

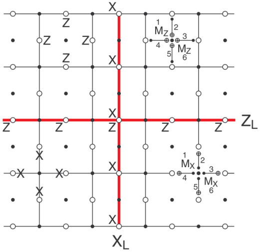


FIG. 1 (color online). A surface code logical qubit. Stabilizers $ZZZZ$ ($XXXX$) are associated with the data qubits (open circles) around each face (vertex). Syndrome qubits (dots) measure stabilizers using the indicated sequences of gates. Logical operators Z_L, X_L connect opposing boundaries.

From [PRA **86**, 032324 (2012)]

Status of current quantum computers

[https://www.ibm.com/quantum/technology]

Quantum Computing in the NISQ era and beyond

John Preskill

Institute for Quantum Information and Matter and Walter Burke Institute for Theoretical Physics,
California Institute of Technology, Pasadena CA 91125, USA
30 July 2018

Noisy Intermediate-Scale Quantum (NISQ) technology will be available in the near future. Quantum computers with 50-100 qubits may be able to perform tasks which surpass the capabilities of today's classical digital computers, but noise in quantum gates will limit the size of quantum circuits that can be executed reliably. NISQ devices will be useful tools for exploring many-body quantum physics, and may have other useful applications, but the 100-qubit quantum computer will not change the world right away — we should regard it as a significant step toward the more powerful quantum technologies of the future. Quantum technologists should continue to strive for more accurate quantum gates and, eventually, fully fault-tolerant quantum computing.

[arXiv:1801.00862]

Status of current quantum computers

It's a remarkable claim — one of the most amazing ideas I've encountered in my scientific life — that there is a distinction between problems that are classically hard and problems that are quantumly hard. And it is a compelling challenge to understand better what problems are classically hard but quantumly easy [4, 5]. We should recognize in particular that the power of a quantum computer is not unlimited. We don't expect, for example, that a quantum computer will be able to solve efficiently the hard instances of NP-hard problems like the traveling salesman problem. For such hard combinatorial search problems we probably can't do much better than exhaustively searching for a solution. Quantum computers can speed up exhaustive search [6], but only modestly [7], so NP-hard problems are likely to be quantumly hard as well as classically hard.

[arXiv:1801.00862]

Focus on “quantum advantage”

⋆ What problems are “hard” on classical computers, which might be efficient for quantum computers?

QUANTUM COMPUTING AND THE ENTANGLEMENT FRONTIER

JOHN PRESKILL

Institute for Quantum Information and Matter
California Institute of Technology
Pasadena, CA 91125, USA

Quantum information science explores the frontier of highly complex quantum states, the “entanglement frontier.” This study is motivated by the observation (widely believed but unproven) that classical systems cannot simulate highly entangled quantum systems efficiently, and we hope to hasten the day when well controlled quantum systems can perform tasks surpassing what can be done in the classical world. One way to achieve such “quantum supremacy” would be to run an algorithm on a quantum computer which solves a problem with a super-polynomial speedup relative to classical computers, but there may be other ways that can be achieved sooner, such as simulating exotic quantum states of strongly correlated matter. To operate a large scale quantum computer reliably we will need to overcome the debilitating effects of decoherence, which might be done using “standard” quantum hardware protected by quantum error-correcting codes, or by exploiting the nonabelian quantum statistics of anyons realized in solid state systems, or by combining both methods. Only by challenging the entanglement frontier will we learn whether Nature provides extravagant resources far beyond what the classical world would allow.

Rapporteur talk at the 25th Solvay Conference on Physics
“The Theory of the Quantum World”
Brussels, 19-22 October 2011

[arXiv:1203.5813]

Focus on "quantum advantage"

⋆ **What problems are "hard" on classical computers, which might be efficient for quantum computers?**

How Much Structure Is Needed for Huge Quantum Speedups?

Scott Aaronson*

September 2022

Abstract

I survey, for a general scientific audience, three decades of research into which sorts of problems admit exponential speedups via quantum computers—from the classics (like the algorithms of Simon and Shor), to the breakthrough of Yamakawa and Zhandry from April 2022. I discuss both the quantum circuit model, which is what we ultimately care about in practice but where our knowledge is radically incomplete, and the so-called *oracle* or *black-box* or *query complexity* model, where we've managed to achieve a much more thorough understanding that then informs our conjectures about the circuit model. I discuss the strengths and weaknesses of switching attention to sampling tasks, as was done in the recent quantum supremacy experiments. I make some skeptical remarks about widely-repeated claims of exponential quantum speedups for practical machine learning and optimization problems. Through many examples, I try to convey the "law of conservation of weirdness," according to which every problem admitting an exponential quantum speedup must have some unusual property to allow the amplitude to be concentrated on the unknown right answer(s).

Edited transcript of a rapporteur talk delivered at the 28^{th} Solvay Physics Conference in Brussels, Belgium on May 21, 2022.

[arXiv:2209.06930]

Focus on “quantum advantage”

⋆ What problems are “hard” on classical computers, which might be efficient for quantum computers?

Assessing requirements to scale to practical quantum advantage

M. E. Beverland,[1] P. Murali,[1] M. Troyer,[1] K. M. Svore,[1] T. Hoefler,[2]
V. Kliuchnikov,[1] G. H. Low,[1] M. Soeken,[3] A. Sundaram,[1] and A. Vaschillo[1]

[1] *Microsoft Quantum, Redmond, WA 98052, USA*
[2] *ETH Zurich, Department of Computer Science, Zürich, 8006, Switzerland*
[3] *Microsoft Quantum, Zurich, Switzerland*

(Dated: November 19, 2022)

While quantum computers promise to solve some scientifically and commercially valuable problems thought intractable for classical machines, delivering on this promise will require a *large-scale* quantum machine. Understanding the impact of architecture design choices for a scaled quantum stack for specific applications, prior to full realization of the quantum system, is an important open challenge. To this end, we develop a framework for quantum resource estimation, abstracting the layers of the stack, to estimate resources required across these layers for large-scale quantum applications. Using a tool that implements this framework, we assess three scaled quantum applications and find that hundreds of thousands to millions of physical qubits are needed to achieve practical quantum advantage. We identify three qubit parameters, namely size, speed, and controllability, that are critical at scale to rendering these applications practical. A goal of our work is to accelerate progress towards practical quantum advantage by enabling the broader community to explore design choices across the stack, from algorithms to qubits.

[arXiv:2211.07629]

Sampling from (noisy) random circuits

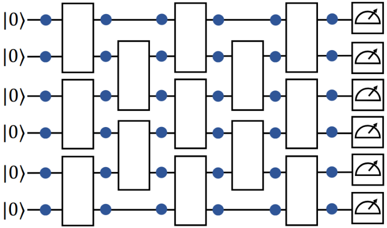
The Computational Complexity of Linear Optics

Scott Aaronson* Alex Arkhipov†

Abstract

We give new evidence that quantum computers—moreover, rudimentary quantum computers built entirely out of linear-optical elements—cannot be efficiently simulated by classical computers. In particular, we define a model of computation in which identical photons are generated, sent through a linear-optical network, then nonadaptively measured to count the number of photons in each mode. This model is not known or believed to be universal for quantum computation, and indeed, we discuss the prospects for realizing the model using current technology. On the other hand, we prove that the model is able to solve sampling problems and search problems that are classically intractable under plausible assumptions.

[arXiv:1011.3245]



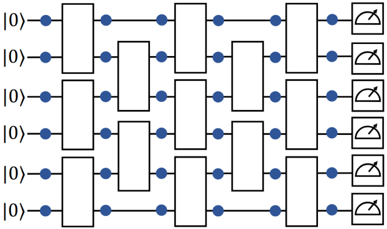
Quantum Sampling Problems, BosonSampling and Quantum Supremacy

A. P. Lund,[1] Michael J. Bremner,[2] and T. C. Ralph[1]

[1] *Centre for Quantum Computation and Communications Technology, School of Mathematics and Physics, The University of Queensland, St Lucia, Queensland 4072, Australia*

[2] *Centre for Quantum Computation and Communications Technology, Centre for Quantum Software and Information, Faculty of Engineering and Information Technology, University of Technology Sydney, NSW 2007, Australia*

[arXiv:1702.03061]

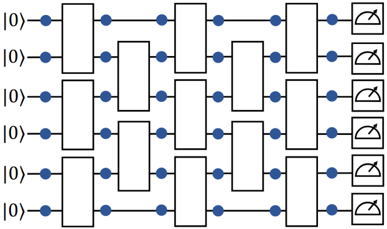


Quantum Supremacy and the Complexity of Random Circuit Sampling

Adam Bouland[1], Bill Fefferman*[1,2], Chinmay Nirkhe[1], and Umesh Vazirani[1]

[1] *Department of Electrical Engineering and Computer Sciences, University of California, Berkeley*
[2] *Joint Center for Quantum Information and Computer Science (QuICS), University of Maryland / NIST*

[arXiv:1803.04402]



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Quantum supremacy using a programmable superconducting processor

Frank Arute, Kunal Arya, Ryan Babbush, Dave Bacon, Joseph C. Bardin, Rami Barends, Rupak Biswas, Sergio Boixo, Fernando G. S. L. Brandao, David A. Buell, Brian Burkett, Yu Chen, Zijun Chen, Ben Chiaro, Roberto Collins, William Courtney, Andrew Dunsworth, Edward Farhi, Brooks Foxen, Austin Fowler, Craig Gidney, Marissa Giustina, Rob Graff, Keith Guerin, … John M. Martinis + Show authors

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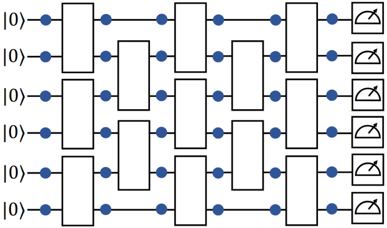
[Nature **574**, 505 (2019)]



Classical simulation

⋆ **What is the boundary between classical and quantum?**

> (3) *No known classical algorithm can simulate a quantum computer.* But perhaps the most persuasive argument we have that quantum computing is powerful is simply that we don't know how to simulate a quantum computer using a digital computer; that remains true even after many decades of effort by physicists to find better ways to simulate quantum systems.

[arXiv:1801.00862]

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[arXiv:1801.00862]

- Roughly: we can simulate low-entangled states using tensor networks.
- Many approximation algorithms have been developed, e.g., in condensed matter theory and quantum chemistry.
- Sufficiently noisy circuits can be classically simulated!

Classical simulation

⋆ **What is the boundary between classical and quantum?**

A polynomial-time classical algorithm for noisy random circuit sampling

Dorit Aharonov[*] Xun Gao[†] Zeph Landau[‡] Yunchao Liu[§] Umesh Vazirani[¶]

Abstract

We give a polynomial time classical algorithm for sampling from the output distribution of a noisy random quantum circuit in the regime of anti-concentration to within inverse polynomial total variation distance. This gives strong evidence that, in the presence of a constant rate of noise per gate, random circuit sampling (RCS) cannot be the basis of a scalable experimental violation of the extended Church-Turing thesis. Our algorithm is not practical in its current form, and does not address finite-size RCS based quantum supremacy experiments.

[arXiv:2211.03999]

Dequantization

⋆ **Quantum algorithms sometimes inspire better classical algorithms!**
(classical algorithms with only polynomial slowdown compared to the quantum one)

A quantum-inspired classical algorithm for recommendation systems

Ewin Tang

May 10, 2019

Abstract

We give a classical analogue to Kerenidis and Prakash's quantum recommendation system, previously believed to be one of the strongest candidates for provably exponential speedups in quantum machine learning. Our main result is an algorithm that, given an $m \times n$ matrix in a data structure supporting certain ℓ^2-norm sampling operations, outputs an ℓ^2-norm sample from a rank-k approximation of that matrix in time $O(\text{poly}(k)\log(mn))$, only polynomially slower than the quantum algorithm. As a consequence, Kerenidis and Prakash's algorithm does not in fact give an exponential speedup over classical algorithms. Further, under strong input assumptions, the classical recommendation system resulting from our algorithm produces recommendations exponentially faster than previous classical systems, which run in time linear in m and n.

The main insight of this work is the use of simple routines to manipulate ℓ^2-norm sampling distributions, which play the role of quantum superpositions in the classical setting. This correspondence indicates a potentially fruitful framework for formally comparing quantum machine learning algorithms to classical machine learning algorithms.

[arXiv:1807.04271]

Dequantization

⋆ **Quantum algorithms sometimes inspire better classical algorithms!**
(classical algorithms with only polynomial slowdown compared to the quantum one)

Quantum principal component analysis only achieves an exponential speedup because of its state preparation assumptions

Ewin Tang*
University of Washington
(Dated: August 10, 2021)

A central roadblock to analyzing quantum algorithms on quantum states is the lack of a comparable input model for classical algorithms. Inspired by recent work of the author [E. Tang, STOC'19], we introduce such a model, where we assume we can efficiently perform ℓ^2-norm samples of input data, a natural analogue to quantum algorithms that assume efficient state preparation of classical data. Though this model produces less practical algorithms than the (stronger) standard model of classical computation, it captures versions of many of the features and nuances of quantum linear algebra algorithms. With this model, we describe classical analogues to Lloyd, Mohseni, and Rebentrost's quantum algorithms for principal component analysis [Nat. Phys. 10, 631 (2014)] and nearest-centroid clustering [arXiv:1307.0411]. Since they are only polynomially slower, these algorithms suggest that the exponential speedups of their quantum counterparts are simply an artifact of state preparation assumptions.

[arXiv:1811.00414]

Dequantization

⋆ **Quantum algorithms sometimes inspire better classical algorithms!**
(classical algorithms with only polynomial slowdown compared to the quantum one)

Revisiting dequantization and quantum advantage in learning tasks

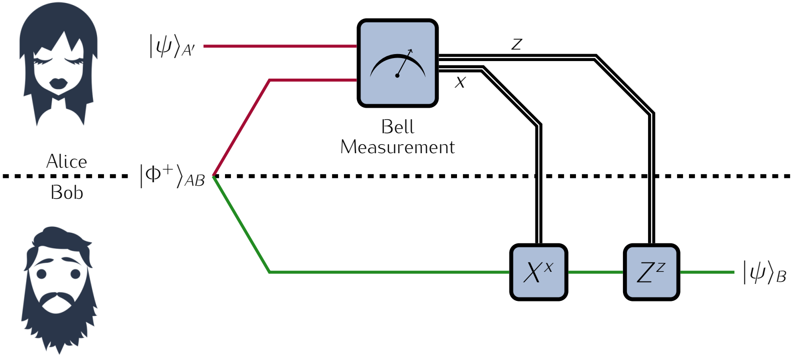
Jordan Cotler,[1,2,*] Hsin-Yuan Huang,[3,4,†] and Jarrod R. McClean[5,‡]

[1] *Harvard Society of Fellows, Cambridge, MA 02138 USA*
[2] *Black Hole Initiative, Cambridge, MA 02138 USA*
[3] *Institute for Quantum Information and Matter, Caltech, Pasadena, CA, USA*
[4] *Department of Computing and Mathematical Sciences, Caltech, Pasadena, CA, USA*
[5] *Google Quantum AI, 340 Main Street, Venice, CA 90291, USA*
(Dated: December 7, 2021)

[arXiv:2112.00811]

Consequences of noise for quantum communication

★ Entangled states are not perfect!



Consequences of noise for quantum communication

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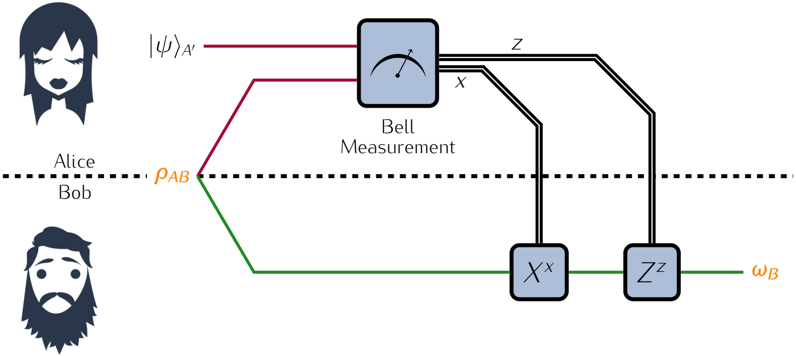

Consequences of noise for quantum communication

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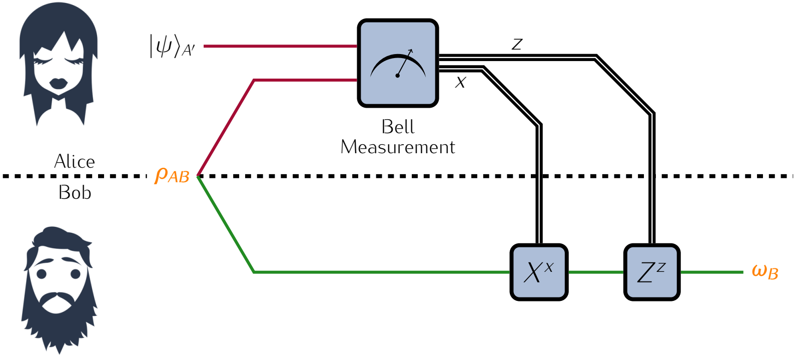
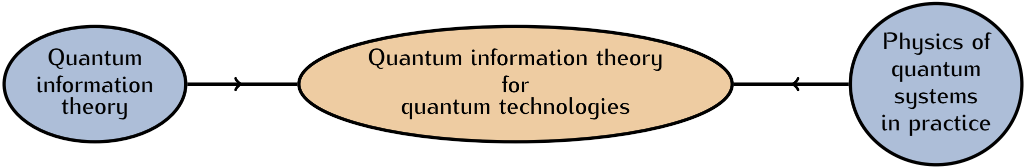


★ Making the entanglement stronger requires *distillation*! An ongoing area of research!



Useful tools

- Mathematical structure of quantum states and quantum evolutions (channels). This allows for physical modeling that is *agnostic* to the specific physical implementation!
- Randomness (e.g., Haar measure).
- Linear and semi-definite programming.