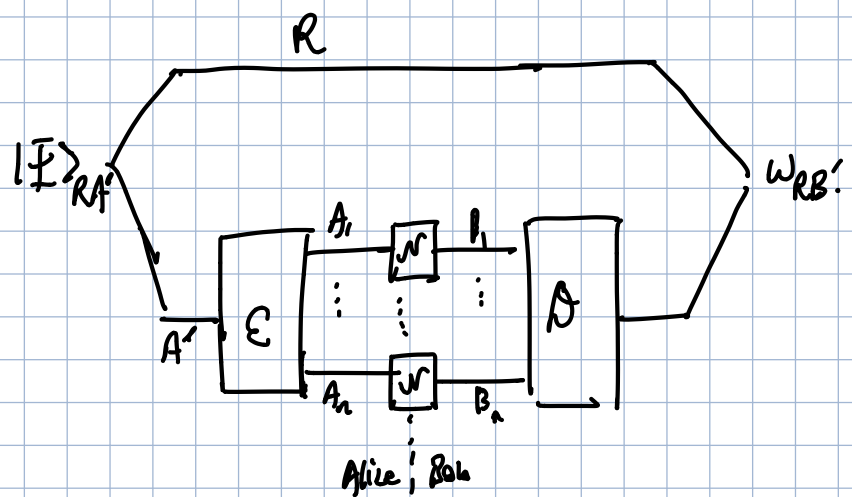
**Desired Goal**: given a channel $\mathcal{N}_{A\to B}$ connecting Alice and Bob, Alice should be able to send any state $|\psi\rangle_A$ to Bob perfectly.

(*) **Stronger**: Alice's system could be in a mixed state and thus entangled with an inaccessible system, so the total state is $|\Psi\rangle_{RA}$, and the goal is to perfectly transmit the A system to Bob. Quantum capacity is the highest rate at which this can be done.

## Resources

- **Unassisted**: only local encoding and decoding allowed; capacity $\boxed{Q(\mathcal{N})}$.




Need $p_{succ}(\mathcal{E},\mathcal{D};\mathcal{N}^{\otimes n}) := \max_{\Psi_{RA}} \langle\Psi|_{RA'} \left(\mathcal{D}_{B^n\to B'} \circ \mathcal{N}^{\otimes n}_{A\to B} \circ \mathcal{E}_{A'\to A^n}\right)(\Psi_{RA'})|\Psi\rangle_{RA'}$

$\equiv F(\mathcal{D}\circ\mathcal{N}^{\otimes n}\circ\mathcal{E})$ to be high.

$$\boxed{p_{err}(\mathcal{E},\mathcal{D};\mathcal{N}^{\otimes n}) = 1 - p_{succ}(\mathcal{E},\mathcal{D};\mathcal{N}^{\otimes n}) = 1 - F(\mathcal{D}\circ\mathcal{N}^{\otimes n}\circ\mathcal{E})}$$

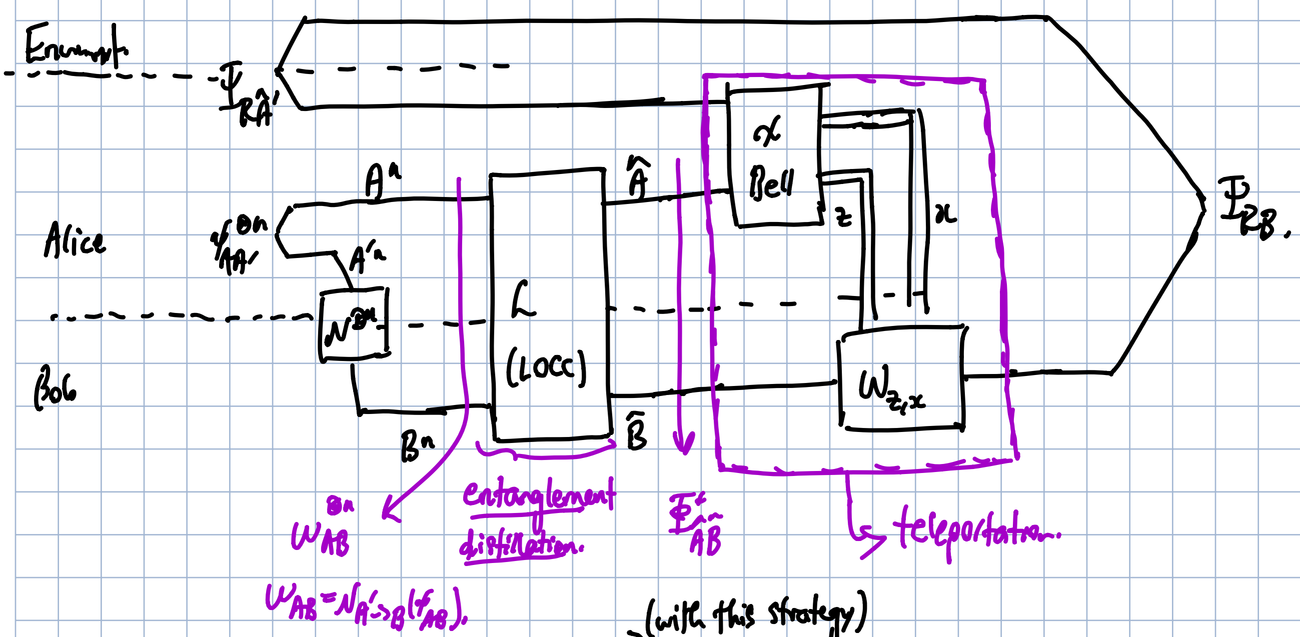
- **LOCC-Assisted**: Local encodings and decodings allowed, and classical comm. both ways b/w channel uses. (i.e., two-way LOCC). Capacity $\boxed{Q^{\leftrightarrow}(\mathcal{N})}$

(*) **Example: Teleportation** → requires only one-way LOCC and a shared maximally entangled state

↳ <u>Modified Goal</u>: Use the channel $\mathcal{N}_{A\to B}$, with LOCC assistance, to create a maximally entangled state b/w Alice and Bob. Then do teleportation.

(*) <u>Achievable strategy, which is optimal</u>: Alice prepares $\psi_{AA'}^{\otimes n}$, sends $A'^n$ through channel to Bob $\Rightarrow \omega_{A^n B^n}^{\otimes n} = \mathcal{N}_{A'\to B}^{\otimes n}(\psi_{AA'}^{\otimes n})$. Then Alice and Bob do two-way entanglement distillation to get max. ent. state. Maximise over all $\psi_{AA'}$, all $n \in \mathbb{N}$, and all two-way LOCC.

→ optimal two-way entanglement distillation is done two-way LOCC on initial state $\rho_{A^n B^n}$ followed by one-way ent. distil. at coh.-inf. rate.




(*) Max. rate of communication (with this strategy) is given by the maximum value of $\log_2 d$, where $d = d_{\hat{A}} = d_{\hat{B}}$ (i.e., dimension of max. ent. state obtained from teleportation).

In the asymptotic setting $(n \to \infty)$, the rate $I(A\rangle B)_\omega$ can be achieved

$$I(A\rangle B) = H(B)_\omega - H(AB)_\omega$$

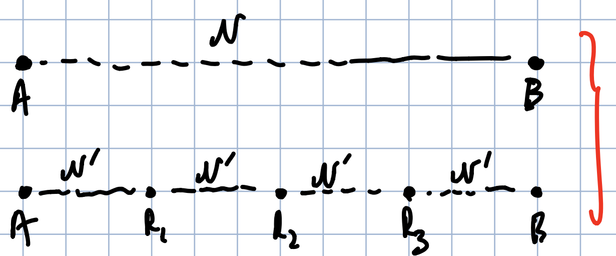
Can optimize over the state $\psi_{AA'} \Rightarrow \boxed{\sup_{\psi_{AA'}} I(A\rangle B)_\omega \equiv I^c(\mathcal{N})}$ can be achieved

↳ Coherent information of $\mathcal{N}$.

(*) It has been shown that $\boxed{Q(\mathcal{N}) = \lim_{k\to\infty} \frac{1}{k} I^c(\mathcal{N}^{\otimes k}).}$ (Can't be computed in general!)

(*) We don't have this problem in ent.-assisted classical comm. b/c the channel mutual inf. is additive

- Repeater-Assisted: Two-way LOCC b/w Alice and Bob and all repeaters.




Main idea: Break up the channel $\mathcal{N}$ into channels $\mathcal{N}'$ that are less noisy.

Use modified goal of creating maximally entangled state b/w A + B.

(*) Achievable Strategy: do entanglement distillation strategy above for each elementary link, then perform entanglement swapping at the repeaters.

Entanglement Swapping is just teleportation!